Existence of solutions of a two-phase free boundary problem

In this talk, we will introduce a variational model describing the morphology of two-phase crystalline systems, that includes the possibility for boundary discontinuities, internal cracks, and external filaments for both phases, and the trade-off between delamination and adhesion at the interface between the two phases. The model consists in a free boundary problem with a configurational energy taking into account not only surface instabilities, but also elastic contributions. By employing the Direct Method of the Calculus of Variation and hence, by identifying a proper topology allowing for both compactness of minimizing sequences and the lower semicontinuity of the energy, we will show the existence of minimizers.

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